1 Fourier Transform

$$x(t) \quad \leftrightarrow \quad X(f)$$

Direct transform $X(f) = \mathscr{F}\{x(t)\} = \int_{-\infty}^{\infty} x(t)\ e^{-j2\pi f t}\ dt$

Inverse transform $x(t) = \mathscr{F}^{-1}\{X(f)\} = \int_{-\infty}^{\infty} X(f)\ e^{j2\pi f t}\ df$

2 Properties

Linearity	$a_1x_1(t) + a_2x_2(t)$	$\leftrightarrow$	$a_1X_1(f) + a_2X_2(f)$
Symmetry	$x(t)$ even	$\leftrightarrow$	$X(f)$ even: $X(-f) = X(f)$
	$x(t)$ odd	$\leftrightarrow$	$X(f)$ odd: $X(-f) = -X(f)$
	$x(t)$ real	$\leftrightarrow$	$X(f)$ Hermitian: $X(-f) = X^*(f)$
	$x(t)$ imaginary	$\leftrightarrow$	$X(f)$ anti-Hermitian: $X(-f) = -X^*(f)$
Time shift	$x(t - t_o)$	$\leftrightarrow$	$e^{-j2\pi f t_0}X(f)$
Frequency translation	$e^{j2\pi f_0 t}x(t)$	$\leftrightarrow$	$X(f - f_o)$
Modulation	$x(t)cos(2\pi f_o t + \phi)$	$\leftrightarrow$	$\frac{1}{2}[e^{j\phi}X(f - f_o) + e^{-j\phi}X(f + f_o)]$
Scaling	$x(at)$	$\leftrightarrow$	$\frac{1}{\lvert a\rvert}X(\frac{f}{a})$
Duality	$X(t)$	$\leftrightarrow$	$x(-f)$
Differentiation	$\frac{d^n x(t)}{dt^n}$	$\leftrightarrow$	$(j2\pi f)^n X(f)$
Multiplication by t^n	$t^n x(t)$	$\leftrightarrow$	$\frac{1}{(-j2\pi)^n}\frac{d^n X(f)}{df^n}$
Integration	$\int_{-\infty}^{t} x(\tau)d\tau$	$\leftrightarrow$	$\frac{1}{j2\pi f}X(f) + \frac{1}{2}X(0)\delta(f)$
Convolution	$x(t) * y(t)$	$\leftrightarrow$	$X(f)Y(f)$
Multiplication	$x(t)y(t)$	$\leftrightarrow$	$X(f) * Y(f)$
Parseval's theorem	$\int_{-\infty}^{\infty} x(t)y^*(t)dt$	$=$	$\int_{-\infty}^{\infty} X(f)Y^*(f)df$
	$\int_{-\infty}^{\infty} \lvert x(t)\rvert^2 dt$	$=$	$\int_{-\infty}^{\infty} \lvert X(f)\rvert^2 df$

3 Basic transforms

Function	$x(t)$	$X(f)$
Impulse	$\delta(t)$	1
Constant	1	$\delta(f)$
Signum	$\mathrm{sign}(t)$	$\frac{1}{j\pi f}$
Step	$u(t)$	$\frac{1}{j2\pi f} + \frac{1}{2}\delta(f)$
Phasor	$e^{j(2\pi f_o t+\phi)}$	$e^{j\phi}\delta(f-f_o)$
Cosinusoid	$\cos(2\pi f_o t+\phi)$	$\frac{1}{2}[e^{j\phi}\delta(f-f_o)+e^{-j\phi}\delta(f+f_o)]$
Sinusoid	$\sin(2\pi f_o t+\phi)$	$\frac{1}{2j}[e^{j\phi}\delta(f-f_o)-e^{-j\phi}\delta(f+f_o)]$
Causal exponencial	$e^{-at}u(t) \quad (a>0)$	$\frac{1}{a+j2\pi f}$
Anti-causal exponential	$e^{at}u(-t) \quad (a>0)$	$\frac{1}{a-j2\pi f}$
Symmetric exponential	$e^{-a\lvert t\rvert} \quad (a>0)$	$\frac{2a}{a^2+(2\pi f)^2}$
Rectangular pulse	$\prod(\frac{t}{T})$	$T\ \mathrm{sinc}(Tf)$
Triangular pulse	$\bigwedge(\frac{t}{T})$	$T\ \mathrm{sinc}^2(Tf)$
Sinc	$\mathrm{sinc}(Bt)$	$\frac{1}{B}\prod(\frac{f}{B})$
Sinc squared	$\mathrm{sinc}^2(Bt)$	$\frac{1}{B}\bigwedge(\frac{f}{B})$
Gaussian	$e^{-\pi(at)^2}$	$\frac{1}{a}\ e^{-\pi(\frac{f}{a})^2}$
Infinite impulse train, $f_s = 1/T_s$	$\sum_{k=-\infty}^{\infty}\delta(t-kT_s)$	$f_s\sum_{n=-\infty}^{\infty}\delta(f-nf_s)$
Periodic signal with basic signal $x_b(t)$, $f_0 = 1/T_0$	$\sum_{k=-\infty}^{\infty}x_b(t-kT_0)$	$f_0\sum_{n=-\infty}^{\infty}X_b(nf_0)\ \delta(f-nf_0)$