Introduction to Digital Systems
Mid-semester examination. November 12, 2021

Time limit: 90 minutes.
Exam results will be released on November 22, 2021

1. (6 points) Consider a decimal digit A, that takes values from 0 to 9, encoded in binary with 4 bits $A_3A_2A_1A_0$ using BCD encoding. You want to implement a function that transforms one decimal digit, A, into another, B, by following the next algorithm:

   if $A \leq 5$ then $B = A + 2$, otherwise $B = A - 2$.

   Note that the decimal digit B can be encoded using the three less significant bits $B_2B_1B_0$ of BCD encoding.

   Look at this problem as the design of a Boolean function with inputs $A_3A_2A_1A_0$ and outputs $B_2B_1B_0$.

   a) (1 point) Determine the truth tables.

   b) (3 points) Use the Karnaugh map to simplify the output Boolean functions.

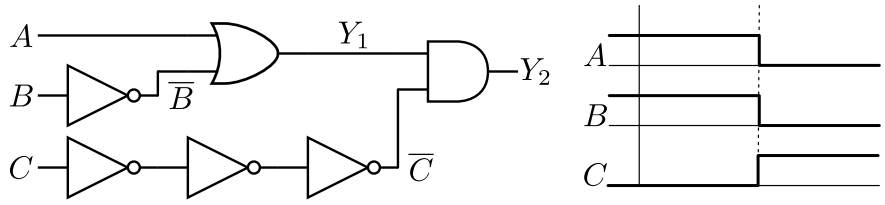
   c) (0.5 points) Implement $B_2B_1B_0$ with logic gates.

   d) (1.5 points) If for some reason $A_3A_2A_1A_0 = 1111$, what values would $B_2B_1B_0$ take?

2. (5 points) Draw the digital waveform of the signals $\overline{B}$, Y_1, $\overline{C}$ and Y_2 when inputs ABC change their value from '110' to '001' considering:

   a) (0.5 points) No propagation delay.

   b) (4.5 points) A propagation delay Δ for each logic gates.



3. (4 points) You work in a company that has just acquired an industrial machine. It has three digital sensors to help decide if the machine can be started safely. When sensor A is '1', YES we can start the machine; otherwise we don't know. When sensor B is '1', YES we can start the machine; otherwise we don't know. When sensor C is '0', NO we can not start the machine; otherwise we don't know. As it is not clear what to do in certain situations, you refer to the instruction manual. This one says that only when the number of YES is greater than the number of NO the machine can be started safely by setting the D signal to '1'.

   Design a circuit that starts the machine safely with the following restrictions.

   a) (0.5 points) Using one 8:1 MUX.

   b) (0.5 points) Using one 4:1 MUX and, if necessary, one NOT gate.

   c) (2 points) Using one 2:1 MUX and, if necessary, some NAND gates.

   d) (1 point) Using one 1:8 DEMUX, and, if necessary, some gates.