Introducció als Sistemes Digitals
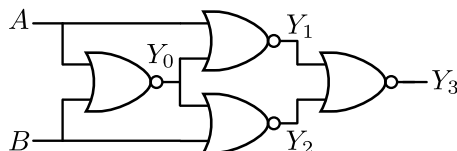# Prova Parcial. 7 de novembre de 2018

Temps per a la resolució: 2 hores. Publicació de resultats: 21 de novembre.

1. (10 points) Cryptography uses ciphers to protect secrets. Substitution ciphers are one of the simplest methods to encrypt a message that started to be used 2500 years ago. The Atbash cipher is a substitution cipher that maps the alphabet to its reverse. We are going to use this cipher to encrypt a decimal digit I that takes values 0-9 coded with 4 bits $I_3 I_2 I_1 I_0$ using the BCD representation. The encrypted decimal digit A is also coded with 4 bits $A_3 A_2 A_1 A_0$ using the BCD representation. The Atbash ciphers encrypts each of these digits $I = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ in the following $A = \{9, 8, 7, 6, 5, 4, 3, 2, 1, 0\}$. For example, $I = 2$ will be encrypted into $A = 7$.

   a) (1 point) Determine the truth table that relates the inputs $I_3 I_2 I_1 I_0$ with the outputs $A_3 A_2 A_1 A_0$.

   b) (2 points) Use the Karnaugh map to simplify the Boolean expression of each output.

   c) (2 points) Implement the Atbash cipher with two-input logic gates AND and OR (and NOT). Count the total number of gates (ignore the NOT gates).

   d) (2 points) Consider the sequence of digits 20181107. Compute the ciphered sequence of digits. Repeat the computation considering that, due to a malfunction, the input I_3 is always high.

   e) (3 points) Finally, design the decipher that will turn back the digit A to its original value I. Use the notation $A_3 A_2 A_1 A_0$ for the input of the decipher, and $O_3 O_2 O_1 O_0$ for its output. Considering the malfunction of the previous question, which will be the sequence of digits O when at the input we have 20181107?

2. (6 punts) Una de les opcions per a construir una porta XNOR a partir de portes NOR és la següent.

   Representeu el cronograma dels senyals Y_0, Y_1, Y_2 i Y_3 quan les entrades AB segueixen la seqüència de valors '01', '00', '10' i '00'. Primer ignoreu l'existència de retard i després considereu un temps de propagació Δ per a totes les portes.

3. (8 punts) Considereu $Y = f(I_2, I_1, I_0)$ que val '1' quan $3 \leq 4I_2 + 2I_1 + I_0 \leq 5$. Implementeu Y amb les següents restriccions.

   a) (1 punt) Usant només un 8:1 MUX.

   b) (2 punts) Usant un 4:1 MUX i portes NOT si cal. Considereu totes les opcions.

   c) (3 punts) Usant un 2:1 MUX i portes lògiques si cal. Considereu totes les opcions.

   d) (2 punts) Usant un 1:8 DEMUX i portes lògiques si cal. Realitzeu dues implementacions diferents.