PROVA PARCIAL - SOLUCIONS

23 de novembre de 2010

1. (a) $M = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$ (b) No reflexiva, no simètrica, antisimètrica, transitiva.

2. (a) $|A \cap B| = |A| - |A \setminus B| = 9 \quad \Rightarrow \quad |B| = |B \setminus A| + |A \cap B| = 21.$

   (b) $|A \cup B| = |A| + |B| - |A \cap B| = 39 \quad \Rightarrow \quad |\overline{A \cup B}| = 11.$

   (c) $|A \cap B| = 9 \quad \Rightarrow \quad |\overline{A \cap B}| = 41.$

3. $D_{63} = \{1, 3, 7, 9, 21, 63\}$

   Cadenes maximals: $(1, 3, 9, 63), (1, 3, 21, 63), (1, 7, 21, 63).$

4. (a) $C_{12,4} C_{36,1} = \binom{12}{4} \binom{36}{1} = 495 \cdot 36 = 17.820$

   (b) $C_{12,4} C_{36,1} + C_{12,5} = \binom{12}{4} \binom{36}{1} + \binom{12}{5} = 17.820 + 792 = 18.612$

5. $CR_{3,12} = \binom{14}{12} = \binom{14}{2} = 91$

6. $(x^2 - 3yz)^5 = \binom{5}{0}(x^2)^5 + \binom{5}{1}(x^2)^4(-3yz) + \binom{5}{2}(x^2)^3(-3yz)^2 + \binom{5}{3}(x^2)^2(-3yz)^3 +$

   $$+ \binom{5}{4} x^2(-3yz)^4 + \binom{5}{5}(-3yz)^5$$

   $(x^2 - 3yz)^5 = x^{10} - 15x^8yz + 90x^6y^2z^2 - 270x^4y^3z^3 + 405x^2y^4z^4 - 243y^5z^5$

7. (a) (i) $\tau \circ \sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 5 & 6 & 2 & 4 & 3 \end{pmatrix}$ (ii) $\sigma \circ \tau = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 5 & 3 & 1 & 2 & 4 \end{pmatrix}$

   (b) $\sigma = (1\,3) \circ (2\,6) \circ (3\,5) \circ (4\,5) \quad \Rightarrow \quad sig(\sigma) = (-1)^4 = +1$

8. (a) $f(x, y, z) = \overline{x}\,\overline{y}\,z + \overline{x}\,\overline{y}\,\overline{z} + x\,y\,z + x\,\overline{y}\,z + \overline{x}\,y\,z$

   (b) $f(x, y, z) = (\overline{x} + y + z)(\overline{x} + \overline{y} + z)(x + \overline{y} + z)$

9. $f(x, y, z, w) = \overline{w} + y\,\overline{z}$