Systems Engineering

Magnetic Coupling

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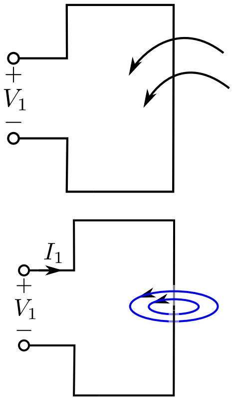
Simple loop



Magnetic Flux

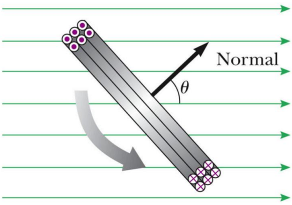
- Lenz's Law $V_1 = \frac{d\phi}{dt}$
- Flux traversing the area
- Proportional to the number of flux lines traversing the area
- Bigger if the area is bigger

Inductance

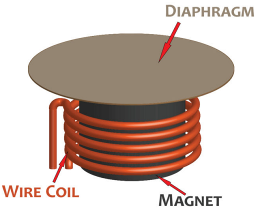
- If there is a current I_1
- $\phi = LI_1$
- $v = L\frac{di_1}{dt}$

Generated Voltages

- Rotating loop in a constant field

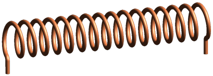


- Dynamic microphone



Inductance of Selected Elements

- Solenoid

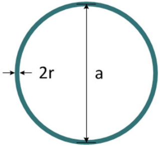


$$B = \mu \frac{Ni}{l}$$

$$V = N\frac{d\phi}{dt} = \{\phi = AB\} = NA\mu\frac{N}{l}\frac{di}{dt} \Rightarrow L = \mu\frac{N^2A}{l}$$

Inductance of Selected Elements /2

- Circular loop



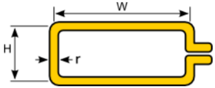
$$L = \mu a(ln\frac{8a}{r} - 2)$$

  - single 1mm dia 5mm loop: 5 nH
  - single 1mm dia 10mm loop: 15 nH

Inductance of Selected Elements /3

- Rectangular loop

RECTANGLE LOOP INDUCTANCE CALCULATOR



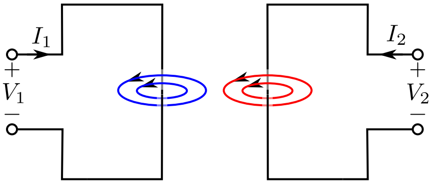
INPUTS

Number of Turns	**N**	1	
Loop Width	**W**	100	mm
Loop Height	**H**	10	mm
Wire Diameter	**d**	1	mm
Relative Permeability	**µR**	1	

OUTPUT

Inductance: 1.26e-7 H

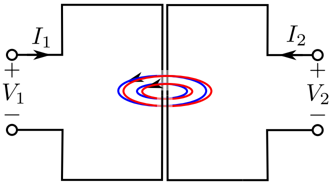
Two loops



What happens

- Far away, independent
- Close together, mutual coupling

Two loops close together



Mutual Coupling Equations

- $V_1 = \frac{d\phi_1}{dt}$
- $V_2 = \frac{d\phi_2}{dt}$

- ϕ_1 has two components: $\phi_1 = \phi_{11} + \phi_{12}$
- $\phi_1 = L_1 i_1 + M_{12} i_2$
  So, we get
- $v_1(t) = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$
- $v_2(t) = M \frac{di_1}{dt} + L_2 \frac{di_2}{dt}$

Note: $M_{12} = M_{21} = M = k\sqrt{L_1 L_2}$ with $k \leq 1$
When $k = 1$ we speak of "perfect coupling"

Laplace domain

Time domain

- $v_1(t) = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$
- $v_2(t) = M \frac{di_1}{dt} + L_2 \frac{di_2}{dt}$

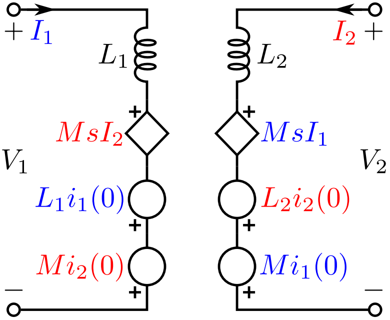
Laplace domain

- $V_1(s) = L_1(sI_1(s) - i_1(0^-)) + M(sI_2(s) - i_2(0^-))$
- $V_2(s) = M(sI_1(s) - i_1(0^-)) + L_2(sI_2(s) - i_2(0^-))$

Equivalent circuit

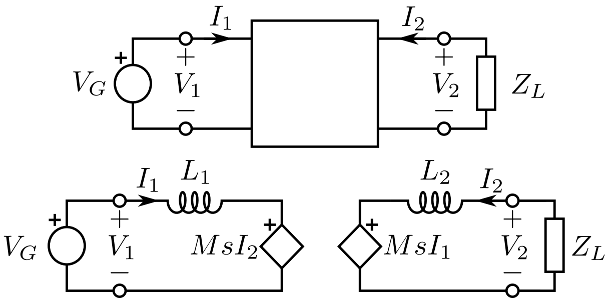
Laplace domain

- $V_1(s) = L_1(sI_1(s) - i_1(0^-)) + M(sI_2(s) - i_2(0^-))$
- $V_2(s) = M(sI_1(s) - i_1(0^-)) + L_2(sI_2(s) - i_2(0^-))$



The dots

A simple circuit



- $L_1 s I_1 = V_G - M s I_2$
- $L_2 s I_2 + Z_L I_2 = -M s I_1$

$$\begin{bmatrix} L_1 s & Ms \\ Ms & L_2 s + Z_L \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_G \\ 0 \end{bmatrix}$$

Solve for V_O

$$I_2 = \frac{-Ms}{L_1 L_2 s^2 + Z_L L_1 s - M^2 s^2} V_G$$

$$V_O = \frac{Z_L M s}{(L_1 L_2 - M^2) s^2 + Z_L L_1 s} V_G$$

If there is perfect coupling, i.e. $k = 1$, $M = \sqrt{L_1 L_2}$ then:

$$V_O = \frac{M}{L_1} V_G = \sqrt{\frac{L_2}{L_1}} V_G = \frac{N_2}{N_1} V_G = n V_G$$

Parameter n is called the turns ratio.

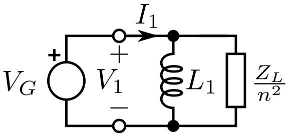
Note that $H(s)$ is a constant, i.e. it is frequency independent!

What is the impedance seen by the generator?

$$I_1 = \frac{L_2 s + Z_L}{(L_1 L_2 - M^2)s^2 + Z_L L_1 s} V_G$$

If $k = 1$ then,

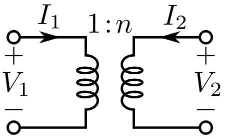
$$I_1 = \frac{L_2}{Z_L L_1} V_G + \frac{V_G}{L_1 s} = \frac{V_G n^2}{Z_L} + \frac{V_G}{L_1 s} = \frac{V_G}{L_1 s} + \frac{V_G}{\frac{Z_L}{n^2}}$$



- First term L_1 depends on the magnetic coupling. No dc!
- Second term depends on the load

An equivalent circuit for the original circuit

Recall the ideal transformer



The original circuit (iff $k = 1$) is equivalent to