Linear Circuits and Systems

Essentials v1.0.

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September 2011

This document describes the essentials of the course Linear Circuits and Systems. Please do not use this *instead* of the recommended bibliography, but in addition to it!

1 Two-ports

Two-ports may be studied at any moment. Their inclusion at the beginning of this course is only to allow for sufficient advances in differential equations solution techniques, taught in Maths.

A two-port is a circuit with four terminals, as depicted in Fig. 1. This figure also emphasizes the *port condition* which requires that the current entering is always the current leaving each port.

1.1 Descriptions

A two-port is described by a set of two equations relating the four variables involved

$$\begin{aligned} f_1(V_1, V_2, I_1, I_2) = 0 \\ f_2(V_1, V_2, I_1, I_2) = 0 \end{aligned} \tag{1}$$

There are six (C_2^4) possibilities to choose two variables as the independent ones. Depending on which we choose, we get the following

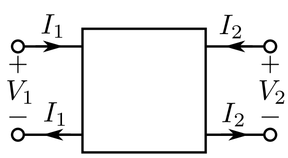

Figura 1: A two-port with the associated variables.

Representation	Independent variables	Dependent variables
Current-controlled	I_1, I_2	V_1, V_2
Voltage-controlled	V_1, V_2	I_1, I_2
Hybrid 1	I_1, V_2	V_1, I_2
Hybrid 2	V_1, I_2	I_1, V_2
Transmission 1	V_2, I_2	V_1, I_1
Transmission 2	V_1, I_1	V_2, I_2

A two-port containing only linear elements and no independent sources is called a linear two-port. In a linear two-port, the six representations may be written as:

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \text{ or } \mathbf{V} = \mathbf{RI} \tag{2}$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \text{ or } \mathbf{I} = \mathbf{GV} \tag{3}$$

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix} \text{ or } \begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \mathbf{H} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix} \text{ or } \begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \mathbf{H_1} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix} \tag{4}$$

$$\begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} H'_{11} & H'_{12} \\ H'_{21} & H'_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ I_2 \end{bmatrix} \text{ or } \begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = \mathbf{H}' \begin{bmatrix} V_1 \\ I_2 \end{bmatrix} \text{ or } \begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = \mathbf{H_2} \begin{bmatrix} V_1 \\ I_2 \end{bmatrix} \tag{5}$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} \text{ or } \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \mathbf{T} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} \text{ or } \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \mathbf{T_1} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} \tag{6}$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} \text{ or } \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \mathbf{T} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} \text{ or } \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \mathbf{T_1} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} \tag{7}$$

Some comments are due regarding the representation chosen for the transmission parameters: The representations in equations (6) and (7) are the most widely used, but there are other alternatives. Note that $-I_2$ is chosen instead of I_2. Since I_2 is the current entering the port through the "+" terminal, if follows that equations (6) and (7) relate the variables with the current *leaving* port 2. This representation is advantageous when dealing with the so-called cascaded connection of two-ports

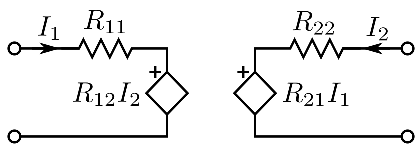

Figura 2: Equivalent circuit of a current-controlled representation.

1.2 Equivalent circuits

The set of equations (2) may be modeled as depicted in figure 2.

A similar representation exists for the current-controlled expression, substituting the Thevenin form with a Norton form. The same idea may be used for finding an equivalent circuit described by the sets of equations (4) and (5).

A direct circuit equivalent for the transmission representations (6) and (7) requires the use of a nullator and a norator, two special one-ports which are seldom used.

1.3 Plotting and physical interpretation of the parameters

The two equations 1 are difficult to plot because of the number of variables involved. However, we may obtain useful plots if we keep one parameter constant.

Considering the Hybrid 1 representation in 5, we may, for instance, write the second equation explicitly:

$$I_2 = H_{21}I_1 + H_{22}V_2 \tag{8}$$

If we let $I_1 = 0$, then we may write

$$H_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0} . \tag{9}$$

This means that parameter H_{22} may be seen as the relation between I_2 and V_2 when port 1 is left open. Equation (8) clearly suggests that V_2 is the stimulus and I_2 the response, but for most practical two-ports there is no difference if the roles are changed. So, to measure H_22 we might connect an ohmmeter to port 2 keeping port 1 open. The inverse of the resistance is directly H_22.

If $I_1 = 0$, the two-port behaves as a conductance of value H_{22}. If we plot his equation, we get a straight line

$$y = mx + c \tag{10}$$

with $m = H_{22}$ and $c = 0$. Now, if $I_1 = 1$, we get a straight line with the same slope, i.e. $m = H_{22}$, and the y intercept point given by $c = H_{21}$. Repeating the process, we get a family of straight lines, parameterized by I_1.

Figures 3 and 4 show a graphical representation of a two-port described by

$$\mathbf{H} = \begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix} \tag{11}$$

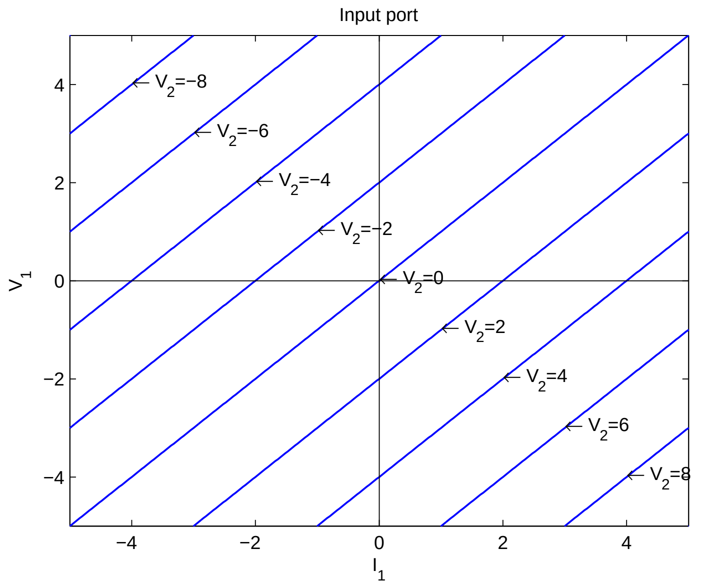

Figura 3: Graphical representation of hybrid 1 parameters. Input port.

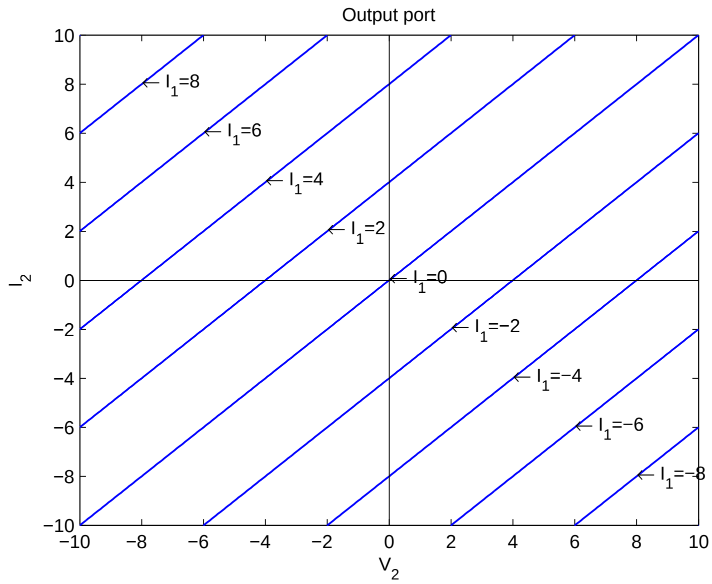

Figura 4: Graphical representation of hybrid 2 parameters. Output port.