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Linear Circuits and Systems

Essentials: Laplace Transformed Circuits

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This document describes the essentials of the course Linear Circuits and Systems. Please do not use this *instead* of the recommended bibliography, but in addition to it!

2 Laplace Transformed Circuits

Last semester, in Circuits Theory, we have learned to analise circuits with only one dynamic element, and that wasn't easy. To analise circuits with several dynamic elements in time domain is complex. In this lesson, we'll take profit from mathematical knowledge, to transform dynamic circuits and to make possible its analysis as we are used to do with resistance circuits.

2.1 Laplace Transforms

You have just learned Laplace Transforms in Maths. Now it's time to apply this knowledge to circuit analysis. There's no need to develop de Laplace transform each time. We usually work with few signals, so the best solution is to know and memorize their Laplace transforms. The table 1 shows the most used signals and its corresponding Laplace transforms.

2.2 Transformed Circuit

At the moment, we know the Laplace Transform. Now it's time to apply this knowledge to circuit analysis. We know that Laplace transforms differential equations into algebraic equations. It will be useful to transform every element and draw directly the transformed circuit. Then we will be able to analyse and study its behaviour directly in s domain. The following figures show, for basic elements, its transformation into the circuit.

Signal	Waveform $f(t)$	Transform $F(s)$
Impulse	$\delta(t)$	1
Step function	$u(t)$	$\frac{1}{s}$
Ramp	$t \cdot u(t)$	$\frac{1}{s^2}$
Exponential	$e^{-\alpha t} \cdot u(t)$	$\frac{1}{s+\alpha}$
Sine	$\sin \omega t \cdot u(t)$	$\frac{\omega}{s^2+\omega^2}$
Cosine	$\cos \omega t \cdot u(t)$	$\frac{s}{s^2+\omega^2}$

Table 1: Signals, waveforms amb Laplace transforms.

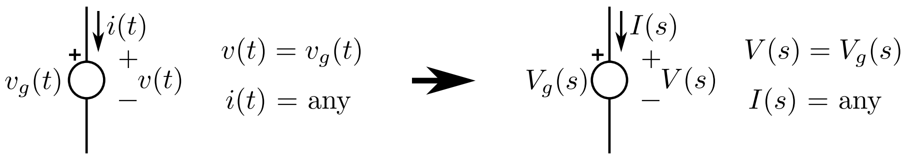

Figure 1: Voltage source and its transformed element

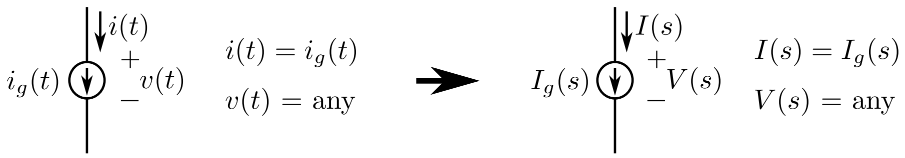

Figure 2: Current source and its transformed element

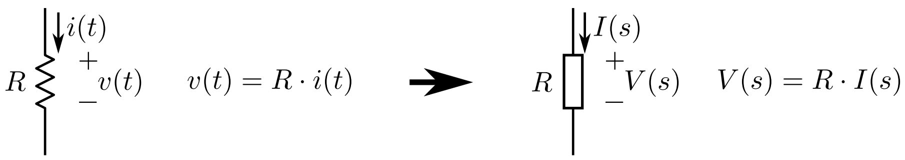

Figure 3: Resistor and its transformed element

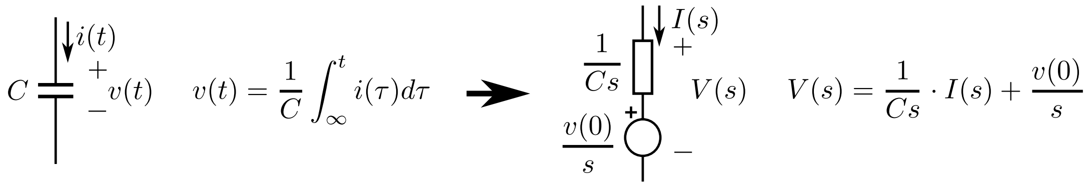

Figure 4: Capacitor and its transformed element

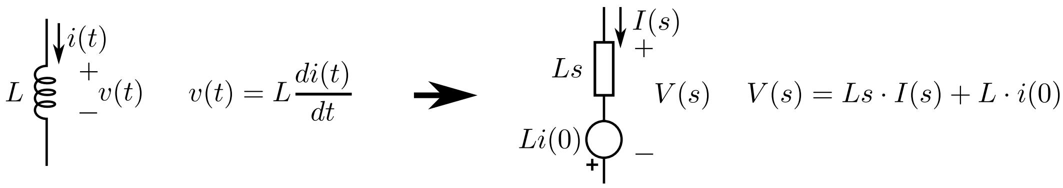

Figure 5: Inductor and its transformed element

2.3 Inverse Laplace Transforms

When we analise a circuit in the transformed domain, we usually find voltages or currents that are ratios of polynomials in s. Before doing the inverse transform, we have to descompose these functions into terms with known inverse transform. We always try to obtain terms with an exponential inverse transform. Sometimes the denominator polynomial has conjugate complex roots. In this case, the exponentials are complex and we have to manipulate the resulting expression to obtain a real signal. To do that, the process to follow is:

After descomposing the function in s domain we obtain the following equation:

$$F(s) = \frac{k}{s-(a+jb)} + \frac{k^*}{s-(a-jb)} \tag{1}$$

The inverse transform of this equation is:

$$f(t) = k \cdot e^{(a+jb)t} + k^* \cdot e^{(a+jb)t} \tag{2}$$

If we write k in its modulus-argument form, equation (2) becomes.

$$f(t) = |k| \cdot e^{j\varphi} \cdot e^{at} \cdot e^{jbt} + |k^*| \cdot e^{-j\varphi} \cdot e^{at} \cdot e^{-jbt} \tag{3}$$

Taking the common terms, we obtain:

$$f(t) = |k| \cdot e^{at} \cdot (e^{j(bt+\varphi)} + e^{-j(bt+\varphi)}) \tag{4}$$

Finally, making use of the identity $e^{jx} + e^{-jx} = 2 \cdot \cos(x)$:

$$f(t) = 2 \cdot |k| \cdot e^{at} \cdot \cos(bt + \varphi) \tag{5}$$

To understand this process, we have to remember how to transform a complex number into its polar form:

If $k = a + jb$ then the module is $|k| = \sqrt{a^2 + b^2}$ and the phase is $\varphi = \arctan \frac{b}{a}$