

Systems Engineering

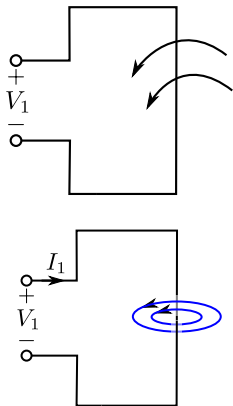
Magnetic Coupling

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Simple loop



Magnetic Flux

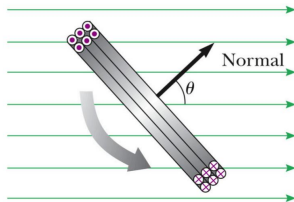
- ▶ Lenz's Law $V_1 = \frac{d\phi}{dt}$
- ▶ Flux traversing the area
- ▶ Proportional to the number of flux lines traversing the area
- ▶ Bigger if the area is bigger

Inductance

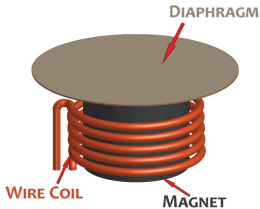
- ▶ If there is a current I_1
- ▶ $\phi = LI_1$
- ▶ $v = L \frac{di_1}{dt}$

Generated Voltages

- ▶ Rotating loop in a constant field

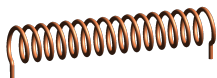


- ▶ Dynamic microphone



Inductance of Selected Elements

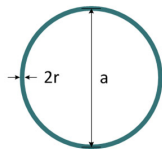
► Solenoid



$$B = \mu \frac{Ni}{l}$$

$$V = N \frac{d\phi}{dt} = \{\phi = AB\} = NA\mu \frac{N}{l} \frac{di}{dt} \Rightarrow L = \mu \frac{N^2 A}{l}$$

Inductance of Selected Elements /2



- ▶ Circular loop

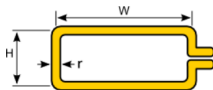
$$L = \mu a \left(\ln \frac{8a}{r} - 2 \right)$$

- ▶ single 1mm dia 5mm loop: 5 nH
- ▶ single 1mm dia 10mm loop: 15 nH

Inductance of Selected Elements /3

► Rectangular loop

RECTANGLE LOOP INDUCTANCE CALCULATOR



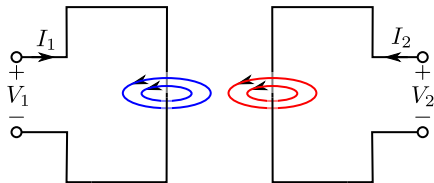
INPUTS

Number of Turns	N	<input type="text" value="1"/>	
Loop Width	W	<input type="text" value="100"/>	<input type="text" value="mm"/>
Loop Height	H	<input type="text" value="10"/>	<input type="text" value="mm"/>
Wire Diameter	d	<input type="text" value="1"/>	<input type="text" value="mm"/>
Relative Permeability	μR	<input type="text" value="1"/>	

OUTPUT

Inductance: 1.26e-7 H

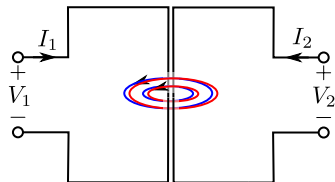
Two loops



What happens

- ▶ Far away, independent
- ▶ Close together, mutual coupling

Two loops close together



Mutual Coupling Equations

- ▶ $V_1 = \frac{d\phi_1}{dt}$
- ▶ $V_2 = \frac{d\phi_2}{dt}$

- ▶ ϕ_1 has two components: $\phi_1 = \phi_{11} + \phi_{12}$

- ▶ $\phi_1 = L_1 i_1 + M_{12} i_2$

So, we get

- ▶ $v_1(t) = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$

- ▶ $v_2(t) = M \frac{di_1}{dt} + L_2 \frac{di_2}{dt}$

Note: $M_{12} = M_{21} = M = k\sqrt{L_1 L_2}$ with $k \leq 1$

When $k = 1$ we speak of “perfect coupling”

Laplace domain

Time domain

$$\blacktriangleright v_1(t) = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$\blacktriangleright v_2(t) = M \frac{di_1}{dt} + L_2 \frac{di_2}{dt}$$

Laplace domain

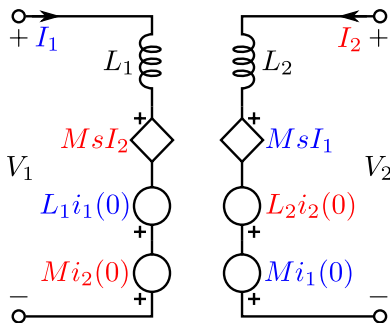
$$\blacktriangleright V_1(s) = L_1(sI_1(s) - i_1(0^-)) + M(sI_2(s) - i_2(0^-))$$

$$\blacktriangleright V_2(s) = M(sI_1(s) - i_1(0^-)) + L_2(sI_2(s) - i_2(0^-))$$

Equivalent circuit

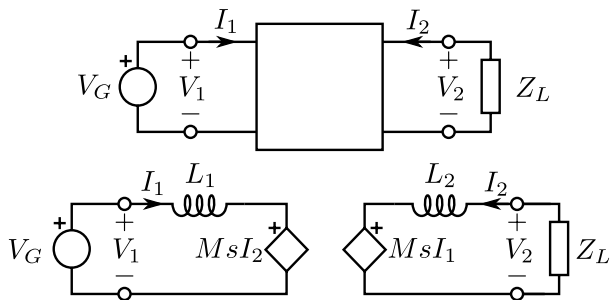
Laplace domain

- ▶ $V_1(s) = L_1(sI_1(s) - i_1(0^-)) + M(sI_2(s) - i_2(0^-))$
- ▶ $V_2(s) = M(sI_1(s) - i_1(0^-)) + L_2(sI_2(s) - i_2(0^-))$



The dots

A simple circuit



- ▶ $L_1 s I_1 = V_G - M s I_2$
- ▶ $L_2 s I_2 + Z_L I_2 = -M s I_1$

$$\begin{bmatrix} L_1 s & M s \\ M s & L_2 s + Z_L \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_G \\ 0 \end{bmatrix}$$

Solve for V_O

$$I_2 = \frac{-Ms}{L_1L_2s^2 + Z_L L_1s - M^2s^2} V_G$$

$$V_O = \frac{Z_L Ms}{(L_1L_2 - M^2)s^2 + Z_L L_1s} V_G$$

If there is perfect coupling, i.e. $k = 1$, $M = \sqrt{L_1L_2}$ then:

$$V_O = \frac{M}{L_1} V_G = \sqrt{\frac{L_2}{L_1}} V_G = \frac{N_2}{N_1} V_G = nV_G$$

Parameter n is called the turns ratio.

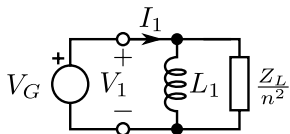
Note that $H(s)$ is a constant, i.e. it is frequency independent!

What is the impedance seen by the generator?

$$I_1 = \frac{L_2 s + Z_L}{(L_1 L_2 - M^2) s^2 + Z_L L_1 s} V_G$$

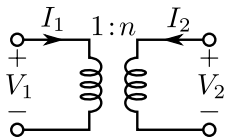
If $k = 1$ then,

$$I_1 = \frac{L_2}{Z_L L_1} V_G + \frac{V_G}{L_1 s} = \frac{n^2}{Z_L} + \frac{V_G}{L_1 s}$$



An equivalent circuit for the original circuit

Recall the ideal transformer



The original circuit (iff $k = 1$) is equivalent to

