

Radiofrequency Circuits and Systems - Useful expressions

Input impedance and propagation constant of an infinite-length transmission line	$Z_0 = \sqrt{\frac{Z}{Y}}, \quad \gamma = \sqrt{ZY}$
Low-loss transmission line	$Z_0 = \sqrt{\frac{L}{C}}, \quad \gamma = \alpha + \frac{s}{V_p}, \quad V_P = \frac{1}{\sqrt{LC}}$
T-parameter matrix of a transmission line of length l and $\theta = \gamma l$	$\mathbf{T} = \begin{bmatrix} \cosh \theta & Z_0 \sinh \theta \\ \frac{1}{Z_0} \sinh \theta & \cosh \theta \end{bmatrix}$
Lossy transmission line	$\theta = \gamma l = \alpha l + \tau s, \quad \tau = \frac{l}{V_p}$
Lossless (ideal) transmission line	$\theta = \gamma l = \tau s$
Wavelength in the sinusoidal steady state	$\lambda = \frac{V_p}{f}$
Reflection coefficient	$\rho = \frac{Z - Z_0}{Z + Z_0}, \quad Z = Z_0 \frac{1 + \rho}{1 - \rho}$
Evolution of ρ along the line	$\rho = \rho_L e^{-2\theta}$
Voltage wave incident on the load in the general problem	$V_{iL} = \frac{1}{2} (1 - \rho_G) \frac{e^{-\theta}}{1 - \rho_G \rho_L e^{-2\theta}} V_G$
Voltage at a distance l from the load	$V = V_{iL} (e^\theta + \rho_L e^{-\theta})$
Current (towards the load) at a distance l from the load	$I = \frac{V_{iL}}{Z_0} (e^\theta - \rho_L e^{-\theta})$
Voltage standing wave ratio	$VSWR = ROE = \frac{1 + \rho }{1 - \rho }$
Attenuation	$A(\text{dB/m}) = \alpha 20 \log_{10} e$
Power transferred to the load	$P_L = \frac{ \bar{V}_{iL} ^2}{2Z_0} (1 - \rho_L ^2)$
Power delivered by the generator	$P_G = \frac{ \bar{V}_{iL} ^2}{2Z_0} (e^{2\alpha l} - \rho_L ^2 e^{-2\alpha l})$
Input reflection coefficient at port 1 of a two-port network loaded with ρ_L at port 2	$\rho_{in} = S_{11} + \frac{S_{12} S_{21} \rho_L}{1 - S_{22} \rho_L}$
Rectangular wave guide	$a = \frac{\lambda}{2 \cos \theta}$

Power density at coordinates (r, θ, φ) with respect to the transmitter	$\mathcal{P} = \frac{P_T}{4\pi r^2} G_T(\theta, \varphi)$
Effective area	$A = \frac{\lambda^2}{4\pi} G$
Friis transmission equation	$P_R = P_T G_T G_R \left(\frac{\lambda}{4\pi r}\right)^2$
Gain of a $\lambda/2$ dipole (arm length = $\lambda/4$)	$G = 1.64 \left(\frac{\cos(\frac{\pi}{2} \cos \theta)}{\sin \theta}\right)^2$
Array of two antennas with phase shift	$FA = [1 + e^{j\alpha} e^{jkd \cos \Phi}]$